UNIVERSITY EXAMINATIONS

SECOND SEMESTER 2023/2024 ACADEMIC YEAR

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (GENERAL)

MATH 122: VECTOR AND MECHANICS 1

STREAM: R   *TIME: 2 HRS*

DAY: WEDNESDAY[8.30A.M – 10.30A.M]   *DATE: 10/04/2024*
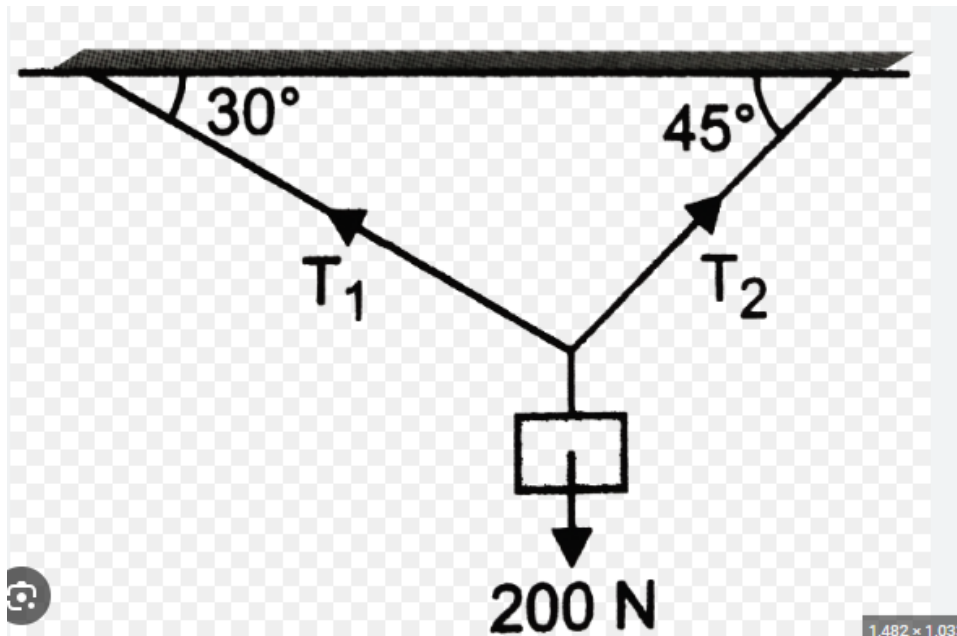
THIS QUESTION PAPER CONSISTS OF SIX (6) PAGES

PLEASE DO NOT OPEN UNTIL THE INVIGILATOR SAYS SO.

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INSTRUCTIONS: **ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

Take g=10N/Kg

G=6.67x10^{-11} Nm^2/Kg^2

QUESTION ONE (30 MARKS)

(a) Define the following terms;

   (i) Linear motion **(2 Marks)**

   (ii) Vector quantity **(2 Marks)**

   (iii)Center of gravity **(2 Marks)**

(b) A force $\vec{F} = 2i + 3j - 7k$ acts on a particle undergoing a displacement $\vec{r} = 4i - 5j + 2k$ Compute the work done by the force. **(3 Marks)**

(c) Find the common velocity after collision **(3 Marks)**
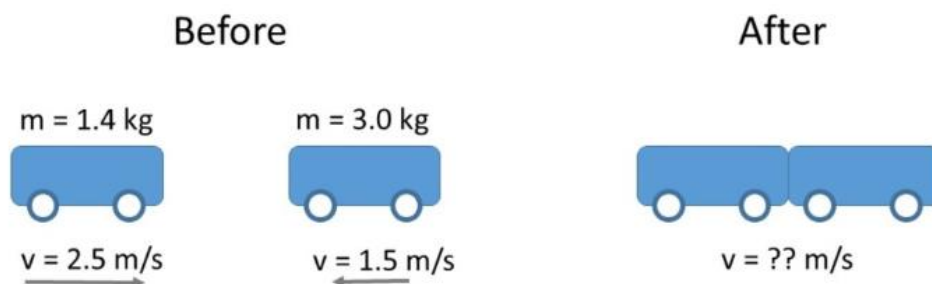
(d) A uniform meter rule balances horizontally on a knife edge placed at the 40 cm mark when a 200g mass is suspended from one end. Determine the weight of the rule **(3 Marks)**

(e) Find the gravitational force between two people each weighing 750N and are standing 150cm apart

(3 Marks)

(f) Determine the Tensions T_1 and T_2 in the strings **(6 Marks)**

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(g) A stone is dropped down a dry well and strikes the bottom after 5 seconds. Neglecting air resistance;

    (i) At what speed does it land on the bottom of the well?   **(3 Marks)**

    (ii) How deep is the well?   **(3 Marks)**

QUESTION TWO (20 MARKS)

(a) An rectangular block of mass 80kg resting on a flat wooden surface requires a minimum force of 560N to start sliding and a force of 480N to keep it moving along the surface at a constant velocity. Determine;

    (i) The coefficient static friction   **(2 Marks)**

    (ii) The coefficient of kinetic friction of the wooden surface   **(3 Marks)**

(b) An object is projected with velocity U and angle of α degrees with the horizontal. If T is the time of flight, H_{max} is the maximum height reached and R is the range of the projectile, show that;

    (i) $T = \dfrac{2U\sin\alpha}{g}$ **(3 Marks)**

    (ii) $H_{max} = \dfrac{U^2\sin^2\alpha}{2g}$ **(3 Marks)**

    (iii) $R = \dfrac{U^2\sin 2\alpha}{g}$ **(3 Marks)**

(c) Decide whether the trajectories of two terrestrial bodies moving with velocities $\vec{u} = 4i + 12j + 16k$ and $\vec{v} = 5i + j - 2k$ are perpendicular or parallel.   **(3 Marks)**

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(d) State the principle of conservation of energy **(3 Marks)**

QUESTION THREE (20 MARKS)

(a) A ball is rolled along the ground. Its initial speed is 16m/s and its deceleration is 4 m/s². How far will it travel? **(3 Marks)**

(b) Calculate the net torque about point O for the two forces applied as shown on the diagram below

(3 Marks)
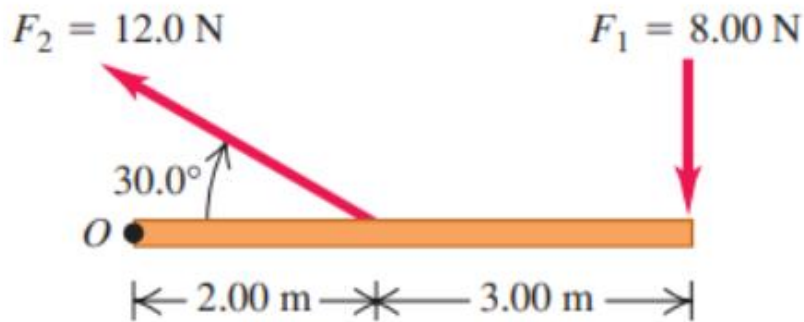
(c) Show that if $\vec{p} = a\vec{i} + b\vec{j} + c\vec{k}$ and $\vec{q} = r\vec{i} + s\vec{j} + t\vec{k}$ then **(4 Marks)**

$$\vec{p} \times \vec{q} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ r & s & t \end{vmatrix}$$

(d) A 20g bullet is fired horizontally at a speed of 400m/s into a 1.5kg block that hangs on a vertical string.

The bullet remains embedded in the block.

(i)   What is the final speed of the bullet and the block? **(3 Marks)**

(ii)   How high will the bullet-block system rise? **(3 Marks)**

(e) Find the angle between the vectors 2i-3j+5k and 4i+5j-k **(4 Marks)**

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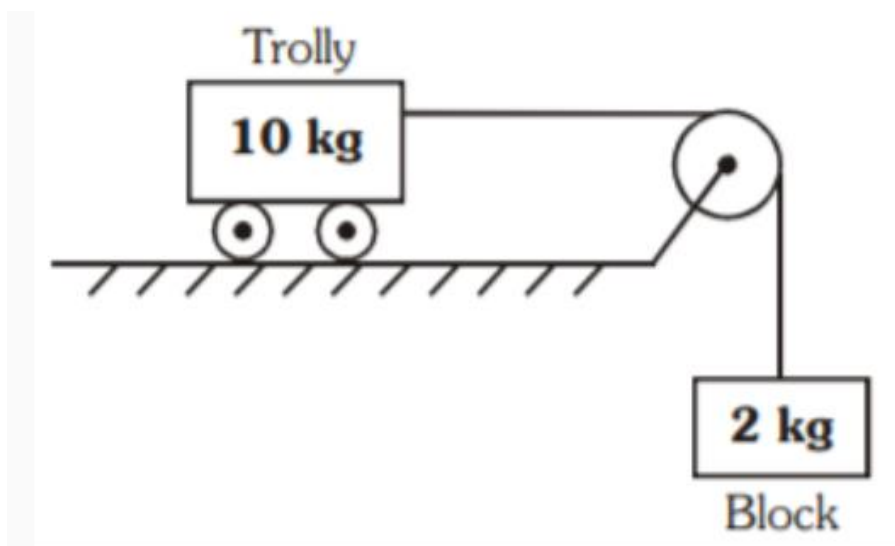**QUESTION FOUR (20 MARKS)**

(a)

    (i) State Newton's second law of motion   **(2 Marks)**

    (ii) A truck of mass 2000kg starts from rest on horizontal rails. Find the speed 3 seconds after starting if the tractive force by the engine is 1000N   **(2 Marks)**

(b) A box with a mass of 12 kg slides along a rough floor with a speed of 5 m/s. If the coefficient of kinetic friction between the box and the floor is 0.23, what is the magnitude of kinetic friction acting on the box?   **(4 Marks)**

(c) A 6.1 kg block slides down a smooth inclined plane whose height is 5 m. What is the velocity of the body at the bottom?   **(4 Marks)**

(d) A referee kicks a ball with a velocity 22m/s at an angle of 45^0 above the horizontal. Assuming there is no air resistance, what is the velocity of the ball 1 second after it was kicked?   **(4 Marks)**

(e) A light string passing over a light smooth pulley carries two masses of 8kg and 12kg at its ends. Find the acceleration and tension in the string if the system is allowed to move from rest.

    (4 Marks)

QUESTION FIVE (20 MARKS)

(a) A large 12 kg fish is swimming through the ocean at 2.5 m/s towards the East. It suddenly sees a smaller 3.0kg fish travelling in the opposite direction at 1.5m/s and decides to have a snack. What is the final velocity of the large fish after enjoying the lunch?   **(4 Marks)**

(b) Calculate the acceleration of the trolley and block system shown below given that coefficient of kinetic friction is 0.05, mass of the string is negligible and no other friction exists   **(4 Marks)**

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Trolly
10 kg

2 kg
Block

(c) Three forces of magnitude 68N, 80N and 54N act on a body in the directions N60^{0}E, S30^{0}W and N45^{0}W respectively. Determine the magnitude and direction of the resultant force **(4 Marks)**

(d) Suppose $\vec{m} = a\vec{i} + b\vec{j} + c\vec{k}$ where a, b and c are non-zero constants.

(i) Write down a vector $\vec{n}$ with magnitude k that is parallel to vector $\vec{m}$ (**2 Marks**)

(ii) Use the definition of the cross product to show that the vectors $\vec{m}$ and $\vec{n}$ are indeed parallel.

(**2 Marks**)

(e) A particle moves along a straight line such that its displacement S meters at any time t seconds is given by $S(t) = t^3 - 6t^2 + 2$. Determine the displacement(s) when;

(i) Velocity is zero. **(2 Marks)**

(ii) Acceleration is 6 ms^{-2} **(2 Marks)**

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